Geometric Algebra Computing

Simple Robot

13.11.2014

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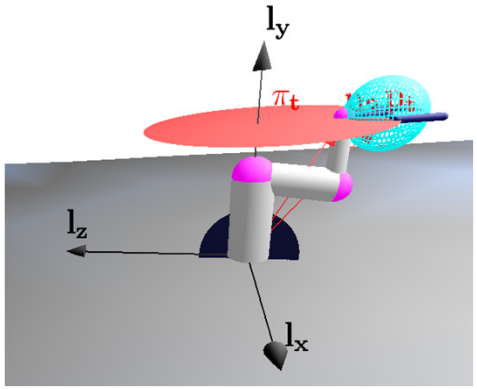

CLUCalc benefits

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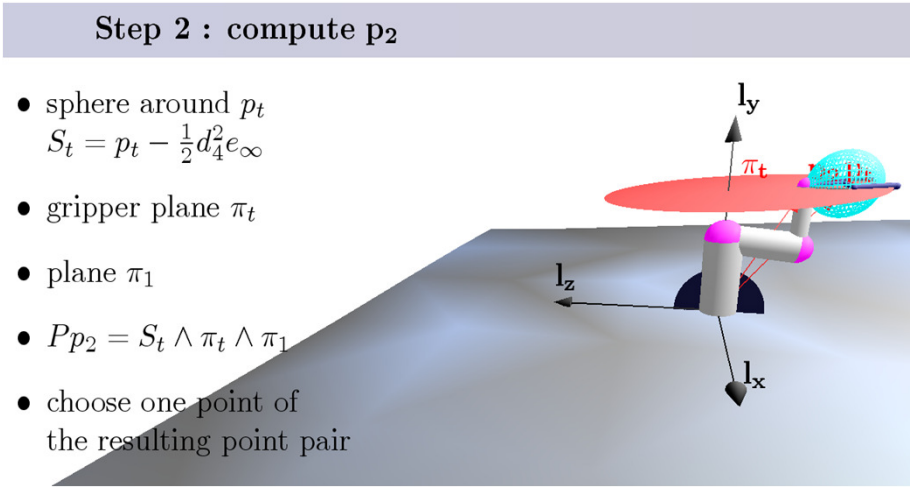
- Visual development of algorithms
- Latex annotations for publications
- Support of presentations ...



Application: Inverse Kinematics of a 5DOF robot

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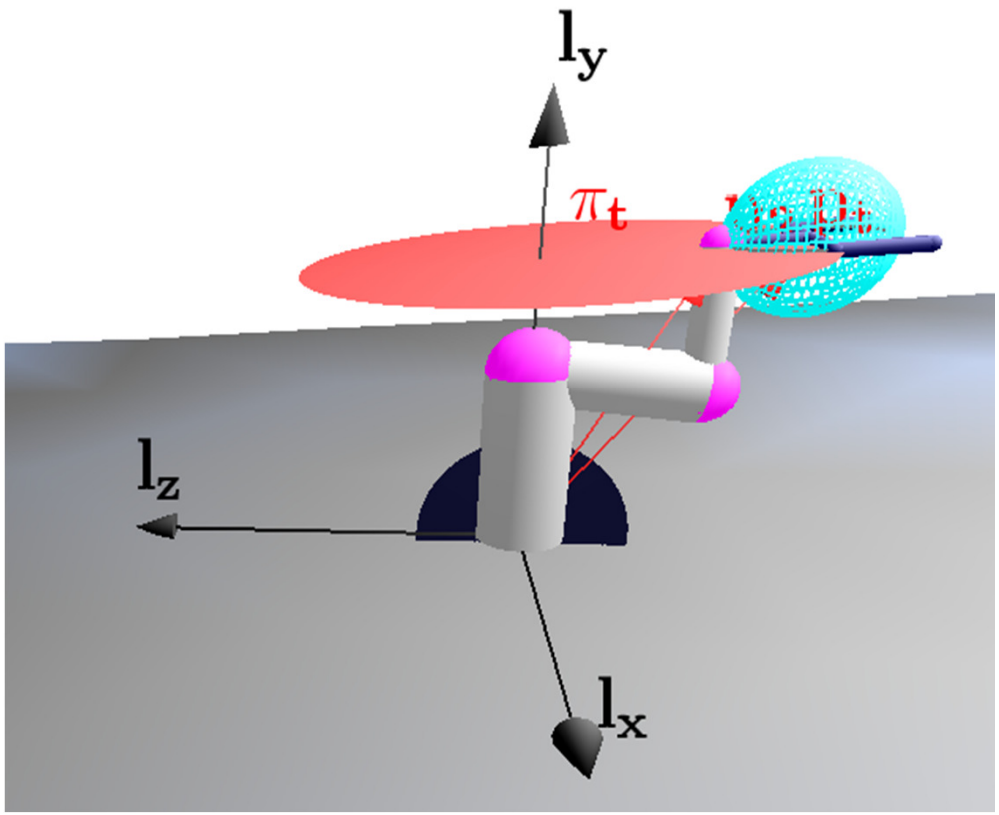
- Easy and intuitive operations with basic geometric entities
  - Entities like spheres and planes
  - Outer product „^“ to intersect



Inverse Kinematics Example

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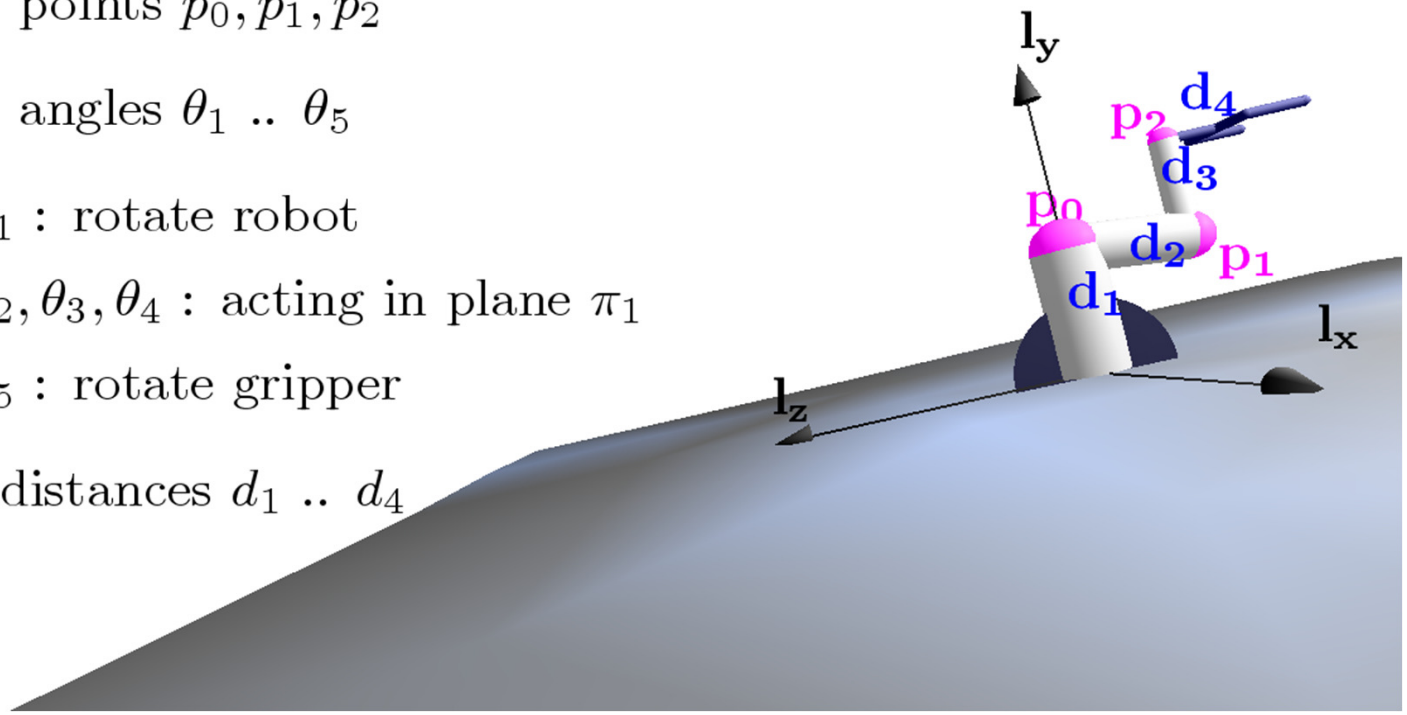
Kinematic chain of a 5DOF-Robot

- 3 joint points p_0, p_1, p_2
- 5 joint angles θ_1 .. θ_5
  - θ_1 : rotate robot
  - $\theta_2, \theta_3, \theta_4$: acting in plane π_1
  - θ_5 : rotate gripper
- 4 link distances d_1 .. d_4



Inverse Kinematics Example

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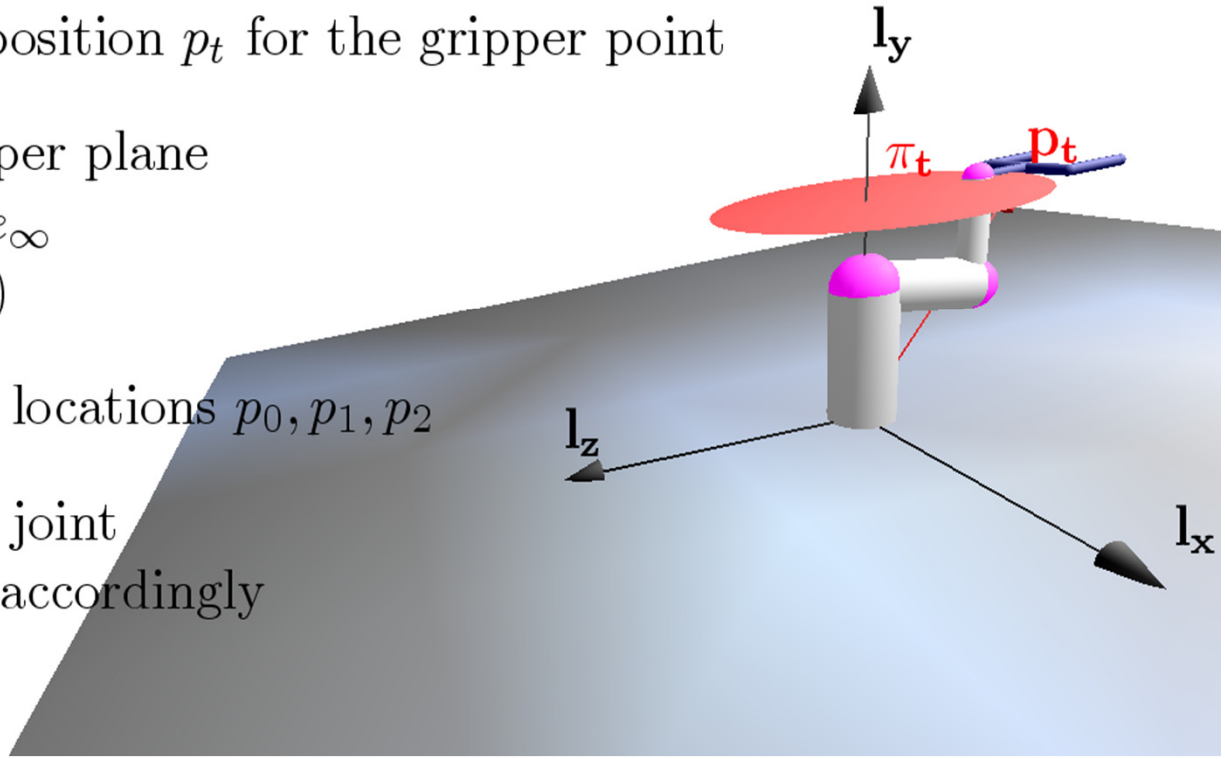
Inverse Kinematics of a 5DOF-Robot

- choose target position p_t for the gripper point
- define the gripper plane
  $\pi_t = e_2 + p_{t,y}\, e_\infty$
  $(\pi = \mathbf{n} + d\, e_\infty)$
- calculate the 3 locations p_0, p_1, p_2
- calculate the 5 joint angles θ_1 .. θ_5 accordingly



Inverse Kinematics Example

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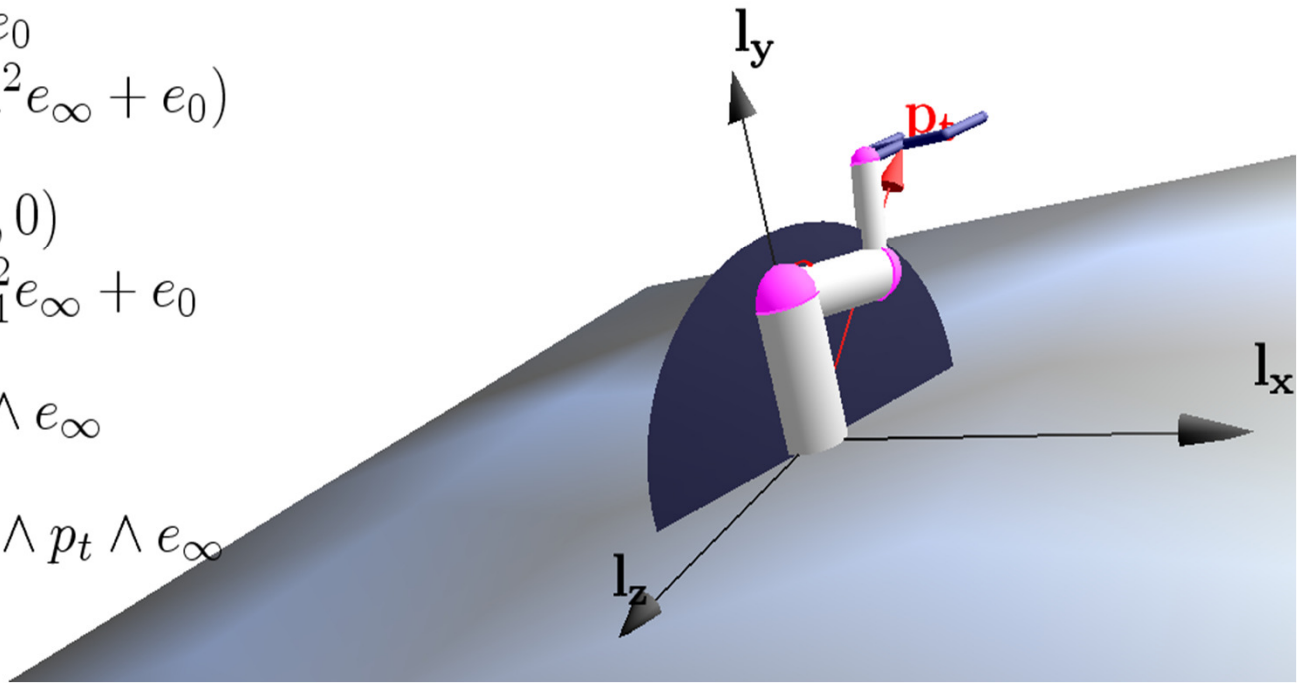
Step 1 : compute $\mathbf{p_0}$

- $P(0,0,0) = e_0$
  $(P = \mathbf{x} + \frac{1}{2}\mathbf{x}^2 e_\infty + e_0)$
- $p_0 = P(0, d_1, 0)$
  $= d_1 e_2 + \frac{1}{2} d_1^2 e_\infty + e_0$
- $l_y^* = e_0 \wedge p_0 \wedge e_\infty$
- $\pi_1^* = e_0 \wedge p_0 \wedge p_t \wedge e_\infty$



Inverse Kinematics Example

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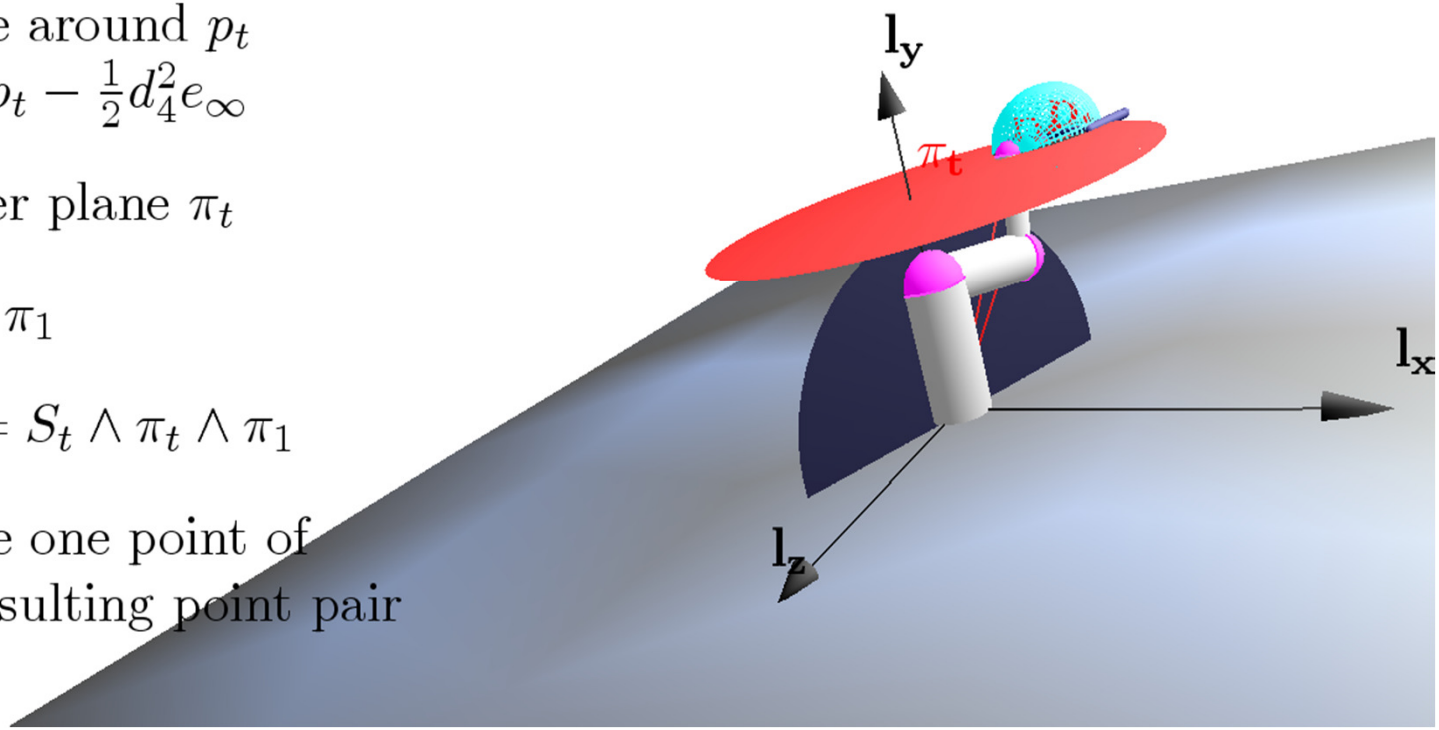
Step 2 : compute p_2

- sphere around p_t
  $S_t = p_t - \frac{1}{2} d_4^2 e_\infty$
- gripper plane π_t
- plane π_1
- $Pp_2 = S_t \wedge \pi_t \wedge \pi_1$
- choose one point of the resulting point pair



Inverse Kinematics Example

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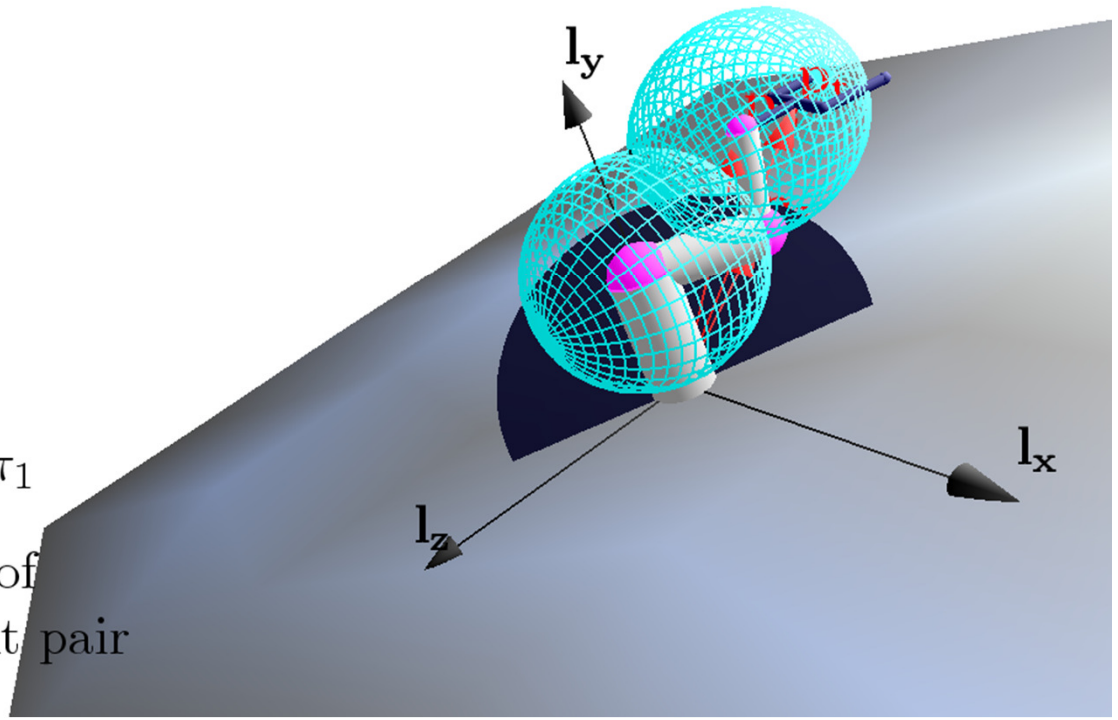
Step 3 : compute $\mathbf{p_1}$

- sphere around p_0
  $S_1 = p_0 - \frac{1}{2} d_2^2 e_\infty$
- sphere around p_2
  $S_2 = p_2 - \frac{1}{2} d_3^2 e_\infty$
- plane π_1
- $Pp_1 = S_1 \wedge S_2 \wedge \pi_1$
- choose one point of the resulting point pair



Inverse Kinematics Example

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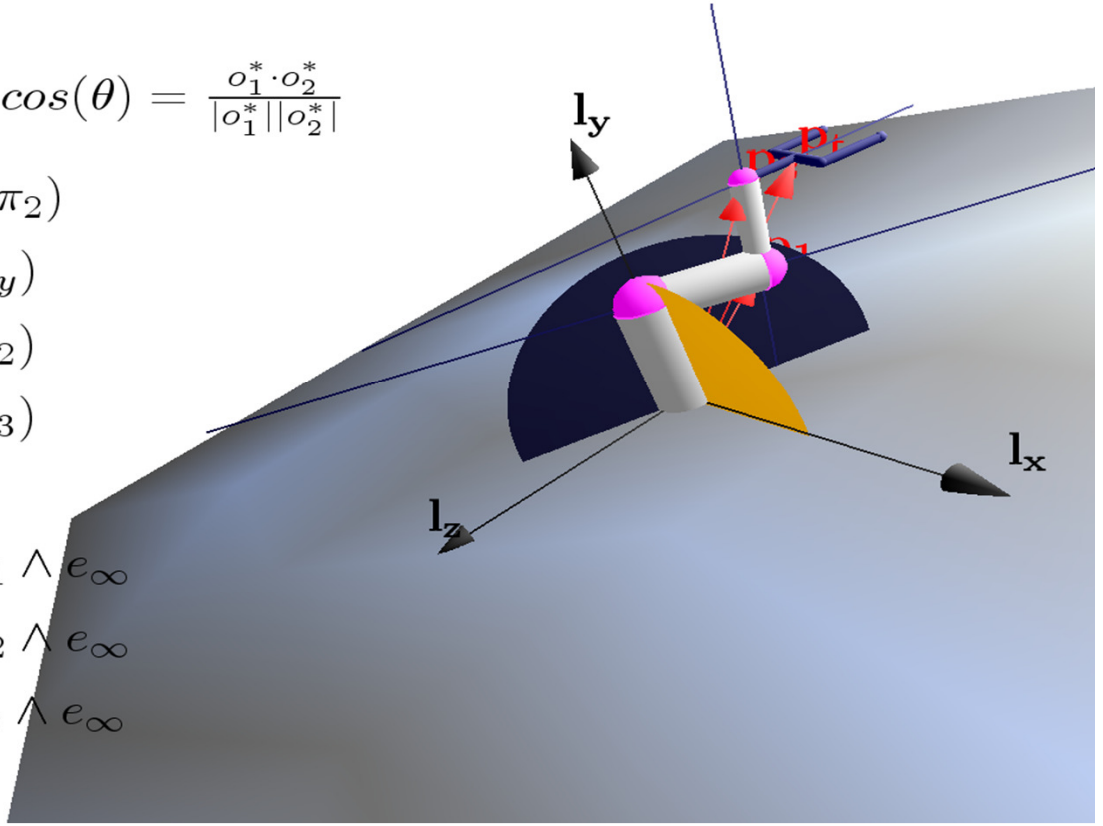
calculate the joint angles $\theta_1..\theta_5$

- with $\angle(o_1, o_2) : cos(\theta) = \frac{o_1^* \cdot o_2^*}{|o_1^*||o_2^*|}$
  - $\theta_1 = \angle(\pi_1, \pi_2)$
  - $\theta_2 = \angle(l_1, l_y)$
  - $\theta_3 = \angle(l_1, l_2)$
  - $\theta_4 = \angle(l_2, l_3)$
  - with
  - $l_1^* = p_0 \wedge p_1 \wedge e_\infty$
  - $l_2^* = p_1 \wedge p_2 \wedge e_\infty$
  - $l_3^* = p_2 \wedge p_t \wedge e_\infty$
  - $\pi_2 = e_3$



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Thanks a lot …