Arbitrary Algebras using GAALOP

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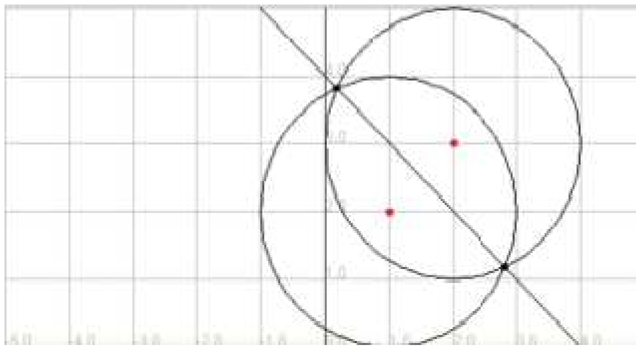


Example: Conic Algebra GAC

- 8-dim GA for the computation with conics

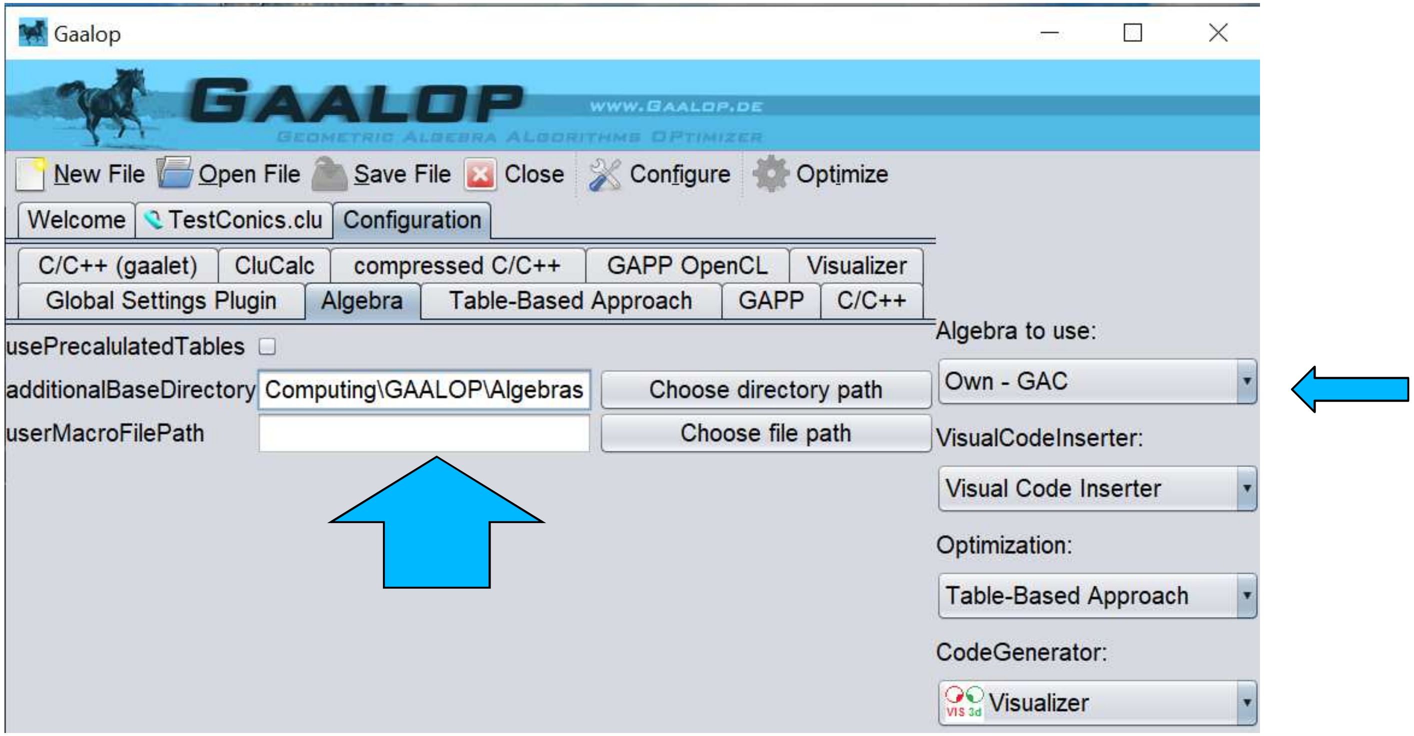
- Reference
  [Geometric Algebra for Conics,
  Jaroslav Hrdina, Ales Navrat and Petr Vasik,
  Advances in Applied Clifford Algebras, 2018]

Arbitrary Algebras using GAALOP

- No fixed limit for the dimension of the algebra
- Note: the multivector of an n-dimensional GA is 2^n dimensional!
- Support of
  - predefined multiplication tables or
  - on-the-fly computation
  - 2d/3d-Visualization
- Integration based on 2 files
  - Definition.csv
  - Macros.clu
- In a specific directory for the own algebras (to be configured in GAALOP)

Integration of Conic Algebra

Add Directory GAC in the additionalBaseDirectory!



and include two specific files for the algebra (definition.csv and macros.clu)

Definition.csv

5 lines in order to define basis vectors …

1,e1,e2,e0p,e0m,e0c,einfp,einfm,einfc

ep1=0.5*einfp-e0p,em1=0.5*einfp+e0p,ep2=0.5*einfm-e0m,em2=0.5*einfm+e0m,ep3=0.5*einfc-e0c,em3=0.5*einfc+e0c

1,e1,e2,ep1,ep2,ep3,em1,em2,em3

e1=1,e2=1,ep1=1,ep2=1,ep3=1,em1=-1,em2=-1,em3=-1

e0p=0.5*em1-0.5*ep1,einfp=em1+ep1,e0m=0.5*em2-0.5*ep2,einfm=em2+ep2,e0c=0.5*em3-0.5*ep3,einfc=em3+ep3

Macros.clu

For basic functionality …

```
createPoint = { cz = _P(3); e0p + _P(1)*e1 + _P(2)*e2 + 0.5*(_P(1)*_P(1) +
_P(2)*_P(2))*einfp + 0.5*(_P(1)*_P(1) - _P(2)*_P(2))*einfm + _P(1)*_P(2)*einfc}

// Define the dual operator * that dualizes A as (*A).
Dual = { _P(1)/(e0p^e0m^e0c^e1^e2^einfp^einfm^einfc)}
...
```

Integration of Conic Algebra

2. Geometric Algebra for Conics

The idea of C. Perwass is to generalize the concept of (two-dimensional) conformal geometric algebra $\mathbb{G}_{3,1}$. In the usual basis $\bar{n}, e_1, e_2, n$, embedding of a plane in $\mathbb{G}_{3,1}$ is given by

$$(x, y) \mapsto \bar{n} + xe_1 + ye_2 + \frac{1}{2}(x^2 + y^2)n.$$

$$C(x, y) = \bar{n}_+ + xe_1 + ye_2 + \frac{1}{2}(x^2 + y^2)n_+ + \frac{1}{2}(x^2 - y^2)n_- + xyn_\times. \quad (2)$$

Integration of Conic Algebra

$$\bar{n}_+, \bar{n}_-, \bar{n}_\times, e_1, e_2, n_+, n_-, n_\times .$$

e0p e0m e0c e1 e2 einfp einfm einfc

Integration of Conic Algebra

$$\bar{n}_+, \bar{n}_-, \bar{n}_\times, e_1, e_2, n_+, n_-, n_\times .$$

e0p e0m e0c e1 e2 einfp einfm einfc

Integration of Conic Algebra

Point

$$C(x, y) = \bar{n}_+ + xe_1 + ye_2 + \frac{1}{2}(x^2 + y^2)n_+ + \frac{1}{2}(x^2 - y^2)n_- + xyn_\times . \quad (2)$$

```
createPoint = {e0p + _P(1)*e1 + _P(2)*e2 + 0.5*(_P(1)*_P(1) +
_P(2)*_P(2))*einfp + 0.5*(_P(1)*_P(1) - _P(2)*_P(2))*einfm +
_P(1)*_P(2)*einfc + _P(3)*0}
```

Integration of Conic Algebra

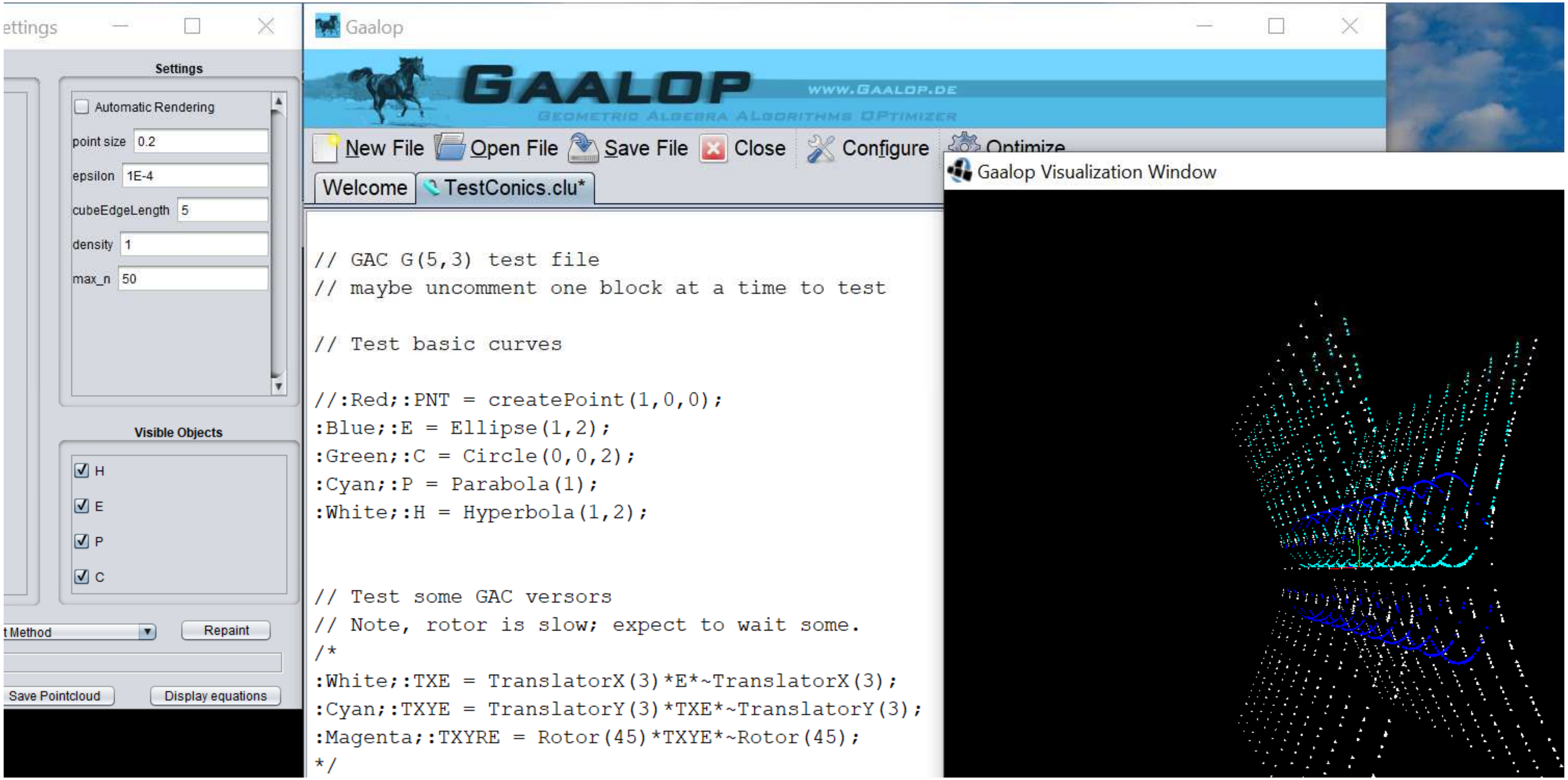
Ellipse

$$
\begin{aligned}
E_I &= (a^2 + b^2)\bar{n}_+ + (a^2 - b^2)\bar{n}_- - a^2 b^2 n_+, \\
H_I &= (a^2 + b^2)\bar{n}_+ + (a^2 - b^2)\bar{n}_- + a^2 b^2 n_+, \\
P_I &= \bar{n}_+ + \bar{n}_- + p e_2.
\end{aligned}
$$

$$\Downarrow$$

Ellipse = { (_P(1)*_P(1) + _P(2)*_P(2))*e0p - (_P(1)*_P(1) - _P(2)*_P(2))*e0m - (_P(1)*_P(1)*_P(2)*_P(2))*einfp}

GAALOP Visualization



GAALOP -> Implicit Functions

```
P=createPoint(x,y,z);
S=e0-0.5*r*r*einf;
?I=S.P;
```

?

GAALOP -> Implicit Functions

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P=createPoint(x,y,z);
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?I=S.P;
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```
I[0] = -z * z - y * y - x * x + r * r; // 1.0
```

?

GAALOP -> Implicit Functions

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P=createPoint(x,y,z);
S=e0-0.5*r*r*einf;
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```

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I[0] = -z * z - y * y - x * x + r * r; // 1.0
```

Sphere around the origin with radius r

GAALOP -> arbitrary visualizations

```
P=createPoint(x,y,z);
S=e0-0.5*r*r*einf;
?I=S.P;
```

```
I[0] = -z * z - y * y - x * x + r * r; // 1.0
```

How can this be directly visualized in Python?

GAALOP -> arbitrary visualizations

Table of Contents

Plotting Module

Introduction

The plotting module allows you to make 2-dimensional and 3-dimensional plots. Presently the plots are rendered using `matplotlib` as a backend. It is also possible to plot 2-dimensional plots using a `TextBackend` if you don't have `matplotlib`.

The plotting module has the following functions:

- plot: Plots 2D line plots.
- plot_parametric: Plots 2D parametric plots.
- plot_implicit: Plots 2D implicit and region plots.
- plot3d: Plots 3D plots of functions in two variables.
- plot3d_parametric_line: Plots 3D line plots, defined by a parameter.
- plot3d_parametric_surface: Plots 3D parametric surface plots.

Thanks a lot

Google Dietmar Hildenbrand