

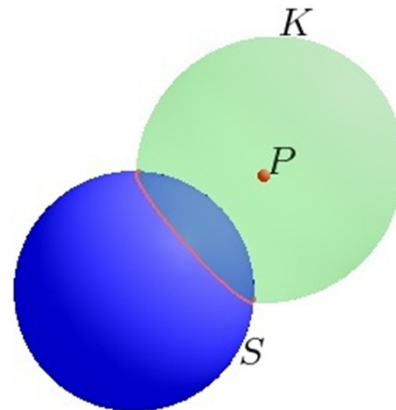
Geometric Algebra Computing



TECHNISCHE
UNIVERSITÄT
DARMSTADT

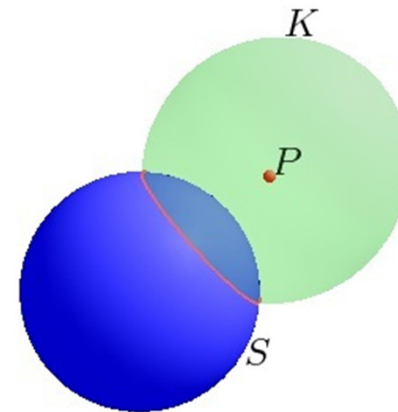
30.10.2014

Dr.-Ing. Dietmar Hildenbrand
Technische Universität Darmstadt



Overview

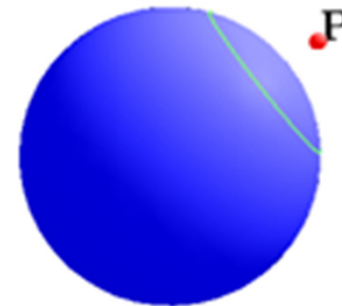
- One GA example (Horizon)
- CLUCalc tutorial (5D-CGA)





Horizon Example

- Compute the horizon circle with
 - observer P
 - Earth S

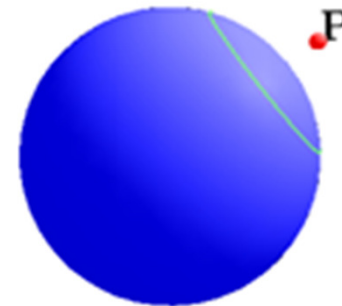


Why horizon example?



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Close to the geometric intuition?
- Simplicity / compactness of algorithms?

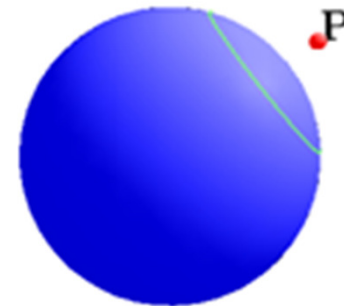


The Solution



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- $P, S, \dots C?$

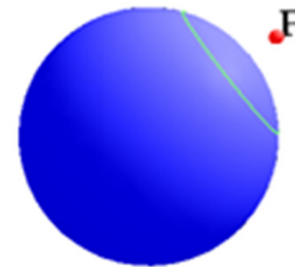


The Solution



TECHNISCHE
UNIVERSITÄT
DARMSTADT

- Which sphere intersects with the sphere S at the horizon circle?

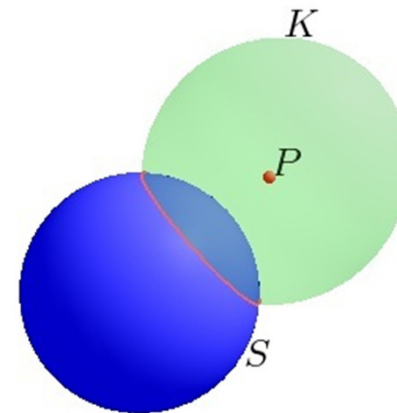


The Solution



TECHNISCHE
UNIVERSITÄT
DARMSTADT

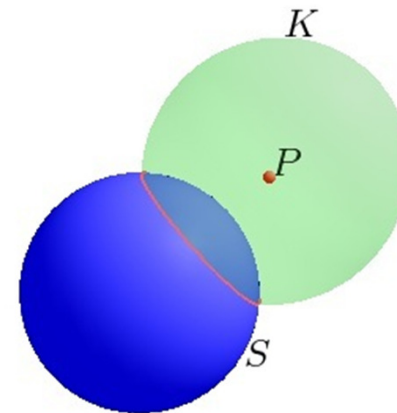
- Which sphere intersects with the sphere S at the horizon circle?
Sphere K with center P and radius such that it touches S





The solution

- Which sphere intersects with the sphere S at the horizon circle?
Sphere K with center P and radius such that it touches S
- The radius of K ?



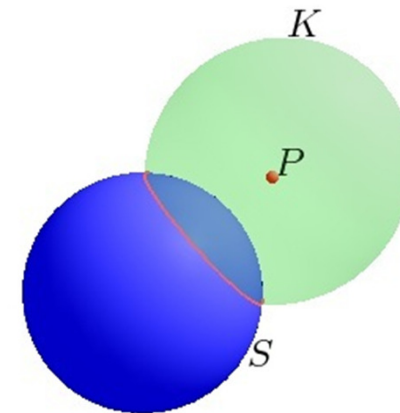
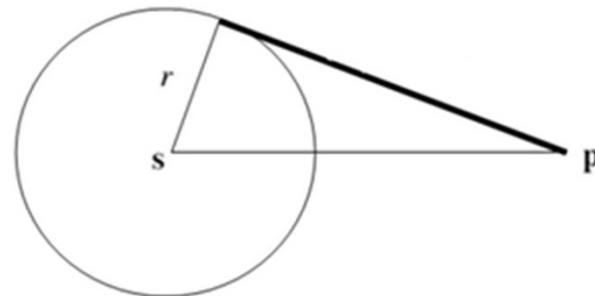


The solution

- Which sphere intersects with the sphere S at the horizon circle?
Sphere K with center P and radius such that it touches S

- The radius a of K ?

$$a^2 = -2S \cdot P$$





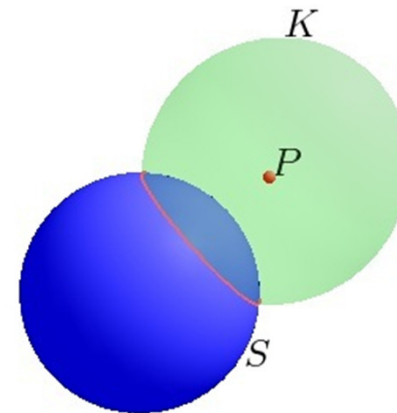
The solution

- Which sphere intersects with the sphere S at the horizon circle?
Sphere K with center P and radius such that it touches S

- The radius a of K ?

$$a^2 = -2S \cdot P$$

- How to describe the sphere K ?





The solution

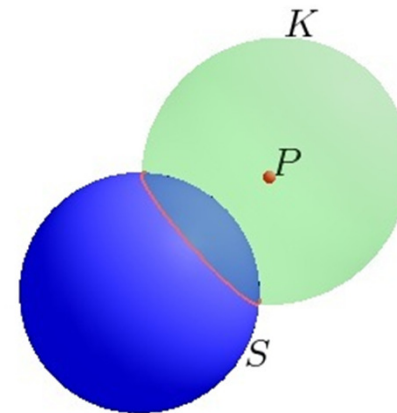
- Which sphere intersects with the sphere S at the horizon circle?
Sphere K with center P and radius such that it touches S

- The radius a of K ?

$$a^2 = -2S \cdot P$$

- How to describe the sphere K ?

$$K = P - \frac{1}{2}a^2 e_\infty = P + (S \cdot P)e_\infty$$





The solution

- Which sphere intersects with the sphere S at the horizon circle?
Sphere K with center P and radius such that it touches S

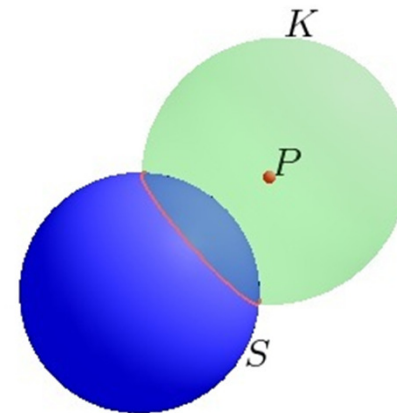
- The radius a of K ?

$$a^2 = -2S \cdot P$$

- How to describe the sphere K ?

$$K = P - \frac{1}{2}a^2 e_\infty = P + (S \cdot P)e_\infty$$

- The formula for the horizon circle C ?





The solution

- Which sphere intersects with the sphere S at the horizon circle?
Sphere K with center P and radius such that it touches S

- The radius a of K ?

$$a^2 = -2S \cdot P$$

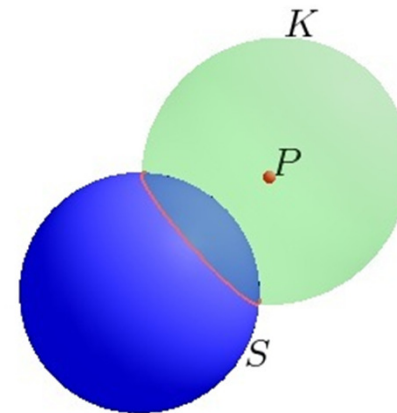
- How to describe the sphere K ?

$$K = P - \frac{1}{2}a^2 e_\infty = P + (S \cdot P)e_\infty$$

- The formula for the horizon circle C ?

$$C = S \wedge K$$

$$C = S \wedge (P + (S \cdot P)e_\infty).$$

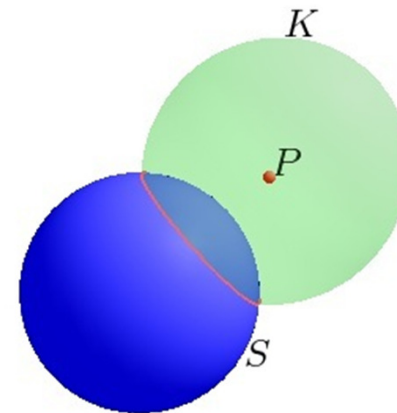




Properties of Geometric Algebra

- geometrically intuitive?
- simple?
- compact?
- role of coordinates?

$$C = S \wedge (P + (S \cdot P)e_{\infty}).$$



Conformal CLUCalc Tutorial



- Chapters 6.4 – 6.5 of „Foundations of Geometric Algebra Computing“

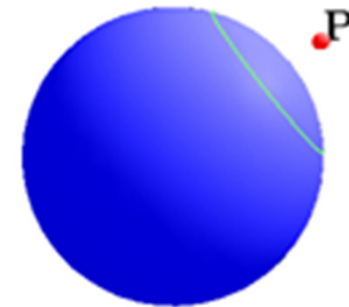
Table 3.2. The two representations (IPNS and OPNS) of conformal geometric entities. The IPNS and OPNS representations are dual to each other, which is indicated by the asterisk symbol.

Entity	IPNS representation	OPNS representation
Point	$P = \mathbf{x} + \frac{1}{2}\mathbf{x}^2 e_\infty + e_0$	
Sphere	$S = P - \frac{1}{2}r^2 e_\infty$	$S^* = P_1 \wedge P_2 \wedge P_3 \wedge P_4$
Plane	$\pi = \mathbf{n} + d e_\infty$	$\pi^* = P_1 \wedge P_2 \wedge P_3 \wedge e_\infty$
Circle	$Z = S_1 \wedge S_2$	$Z^* = P_1 \wedge P_2 \wedge P_3$
Line	$L = \pi_1 \wedge \pi_2$	$L^* = P_1 \wedge P_2 \wedge e_\infty$
Point pair	$Pp = S_1 \wedge S_2 \wedge S_3$	$Pp^* = P_1 \wedge P_2$



Horizontaufgabe CLUCalc-Teil

- Schreiben Sie ein CLUCalc-Programm, welches den Horizontkreis berechnet und visualisiert
- Berechnen und visualisieren Sie den Mittelpunkt des Horizont-Kreises
 - Kugel im Ursprung mit Radius 1
 - Punkt $P=(1,1,0)$
 - Vorschrift in CLUCalc-Notation:
Point = Circle * einf * Circle;
- Wie sind die 3D Koordinaten des Mittelpunkts?
 - Beachte: wie bei homogenen Koordinaten stellen alle skalierten algebraischen Ausdrücke das identische geometrische Objekt dar





TECHNISCHE
UNIVERSITÄT
DARMSTADT

Thanks a lot ...