

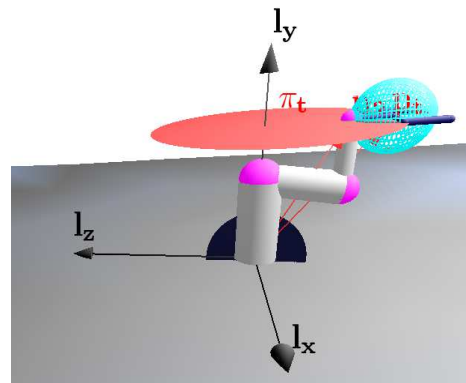
Geometric Algebra Computing

Simple Robot
03.07.2019



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CLUCalc benefits

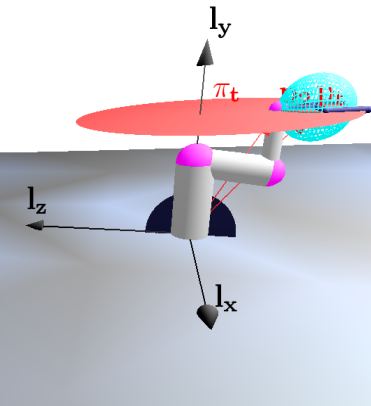


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- Visual development of algorithms
- Latex annotations for publications
- Support of presentations ...

Step 2 : compute p_2

- sphere around p_t
 $S_t = p_t - \frac{1}{2}d_4^2 e_\infty$
- gripper plane π_t
- plane π_1
- $Pp_2 = S_t \wedge \pi_t \wedge \pi_1$
- choose one point of the resulting point pair

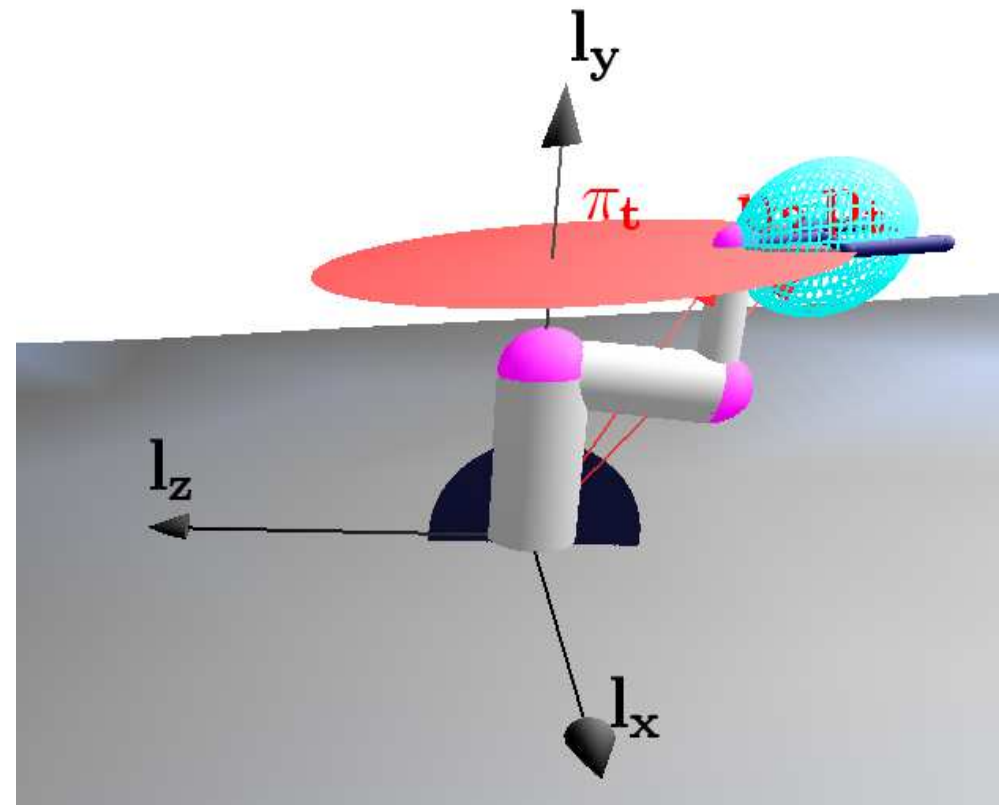


Application: Inverse Kinematics of a 5DOF robot



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- Easy and intuitive operations with basic geometric entities
 - Entities like spheres and planes
 - Outer product „ \wedge “ to intersect

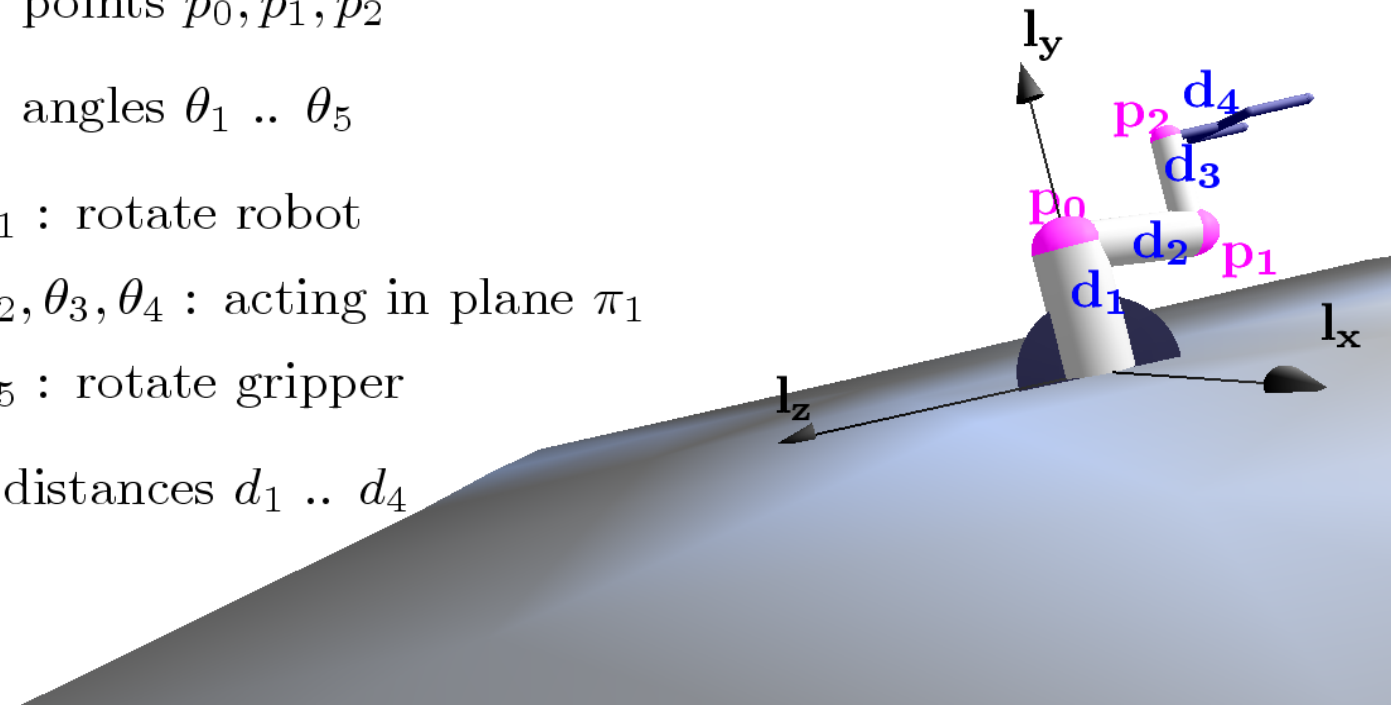


Inverse Kinematics Example



Kinematic chain of a 5DOF-Robot

- 3 joint points p_0, p_1, p_2
- 5 joint angles $\theta_1 .. \theta_5$
 - θ_1 : rotate robot
 - $\theta_2, \theta_3, \theta_4$: acting in plane π_1
 - θ_5 : rotate gripper
- 4 link distances $d_1 .. d_4$



Inverse Kinematics Example



Inverse Kinematics of a 5DOF-Robot

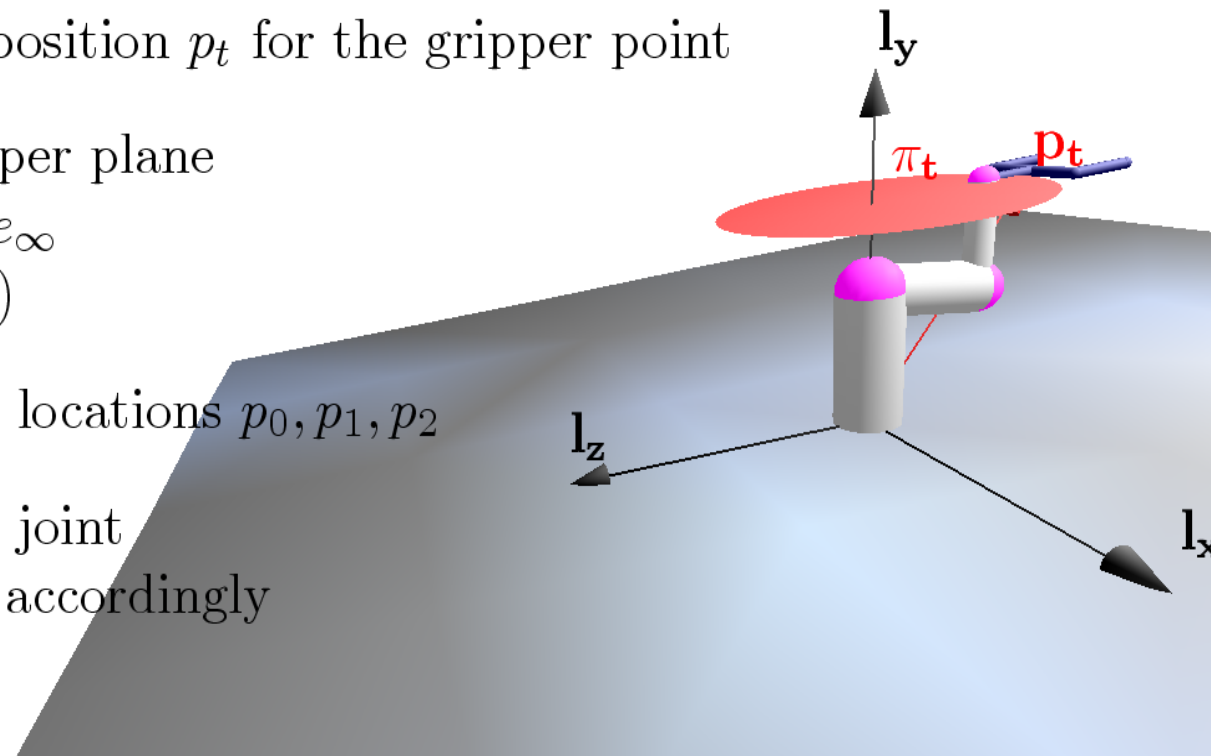
- choose target position p_t for the gripper point

- define the gripper plane

$$\pi_t = e_2 + p_{t,y} e_\infty$$
$$(\pi = \mathbf{n} + d e_\infty)$$

- calculate the 3 locations p_0, p_1, p_2

- calculate the 5 joint angles $\theta_1 \dots \theta_5$ accordingly



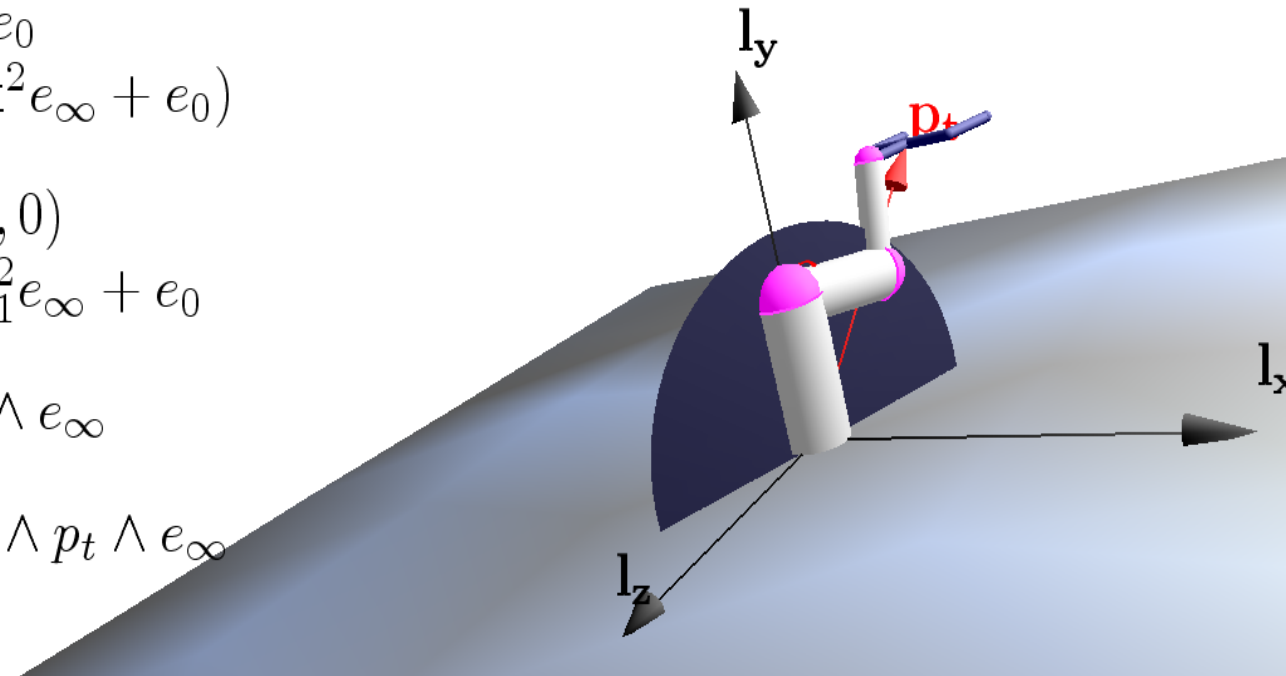
Inverse Kinematics Example



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Step 1 : compute p_0

- $P(0, 0, 0) = e_0$
($P = \mathbf{x} + \frac{1}{2}\mathbf{x}^2 e_\infty + e_0$)
- $p_0 = P(0, d_1, 0)$
 $= d_1 e_2 + \frac{1}{2}d_1^2 e_\infty + e_0$
- $l_y^* = e_0 \wedge p_0 \wedge e_\infty$
- $\pi_1^* = e_0 \wedge p_0 \wedge p_t \wedge e_\infty$

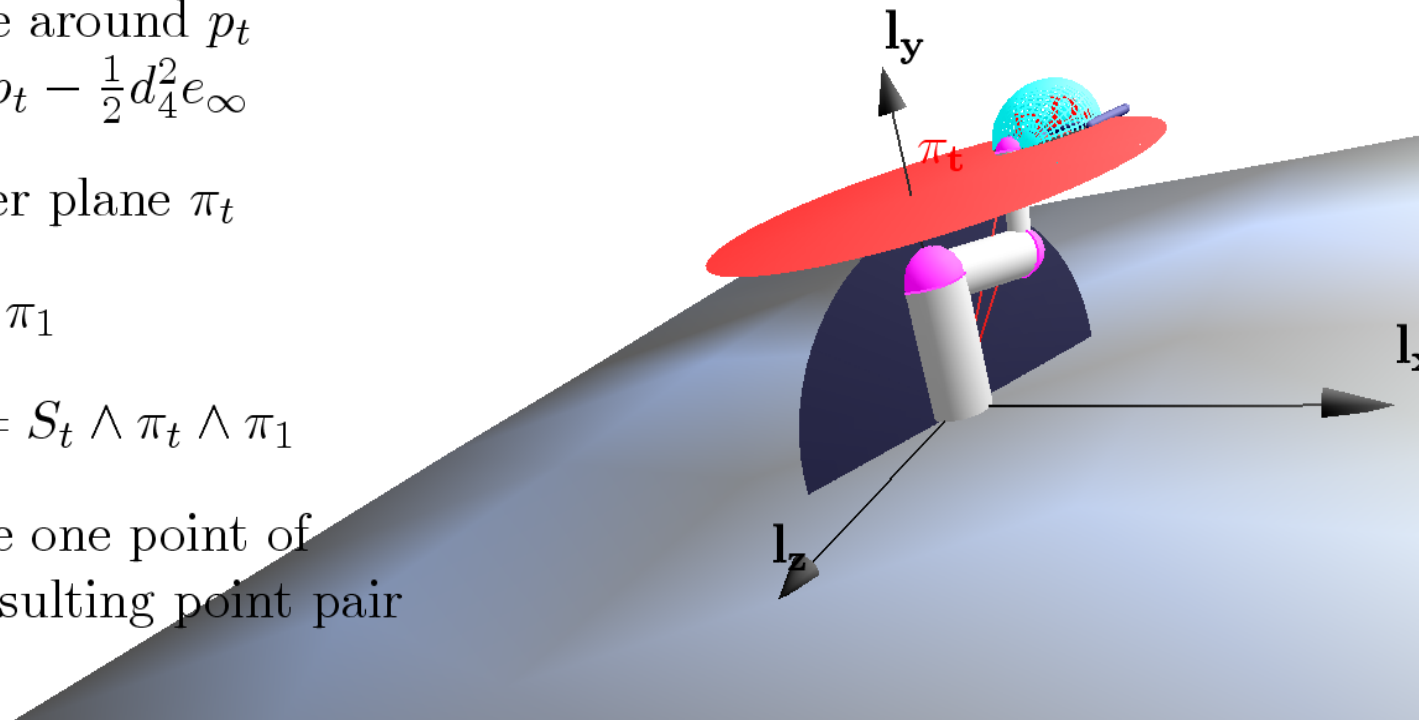


Inverse Kinematics Example



Step 2 : compute p_2

- sphere around p_t
 $S_t = p_t - \frac{1}{2}d_4^2 e_\infty$
- gripper plane π_t
- plane π_1
- $Pp_2 = S_t \wedge \pi_t \wedge \pi_1$
- choose one point of the resulting point pair



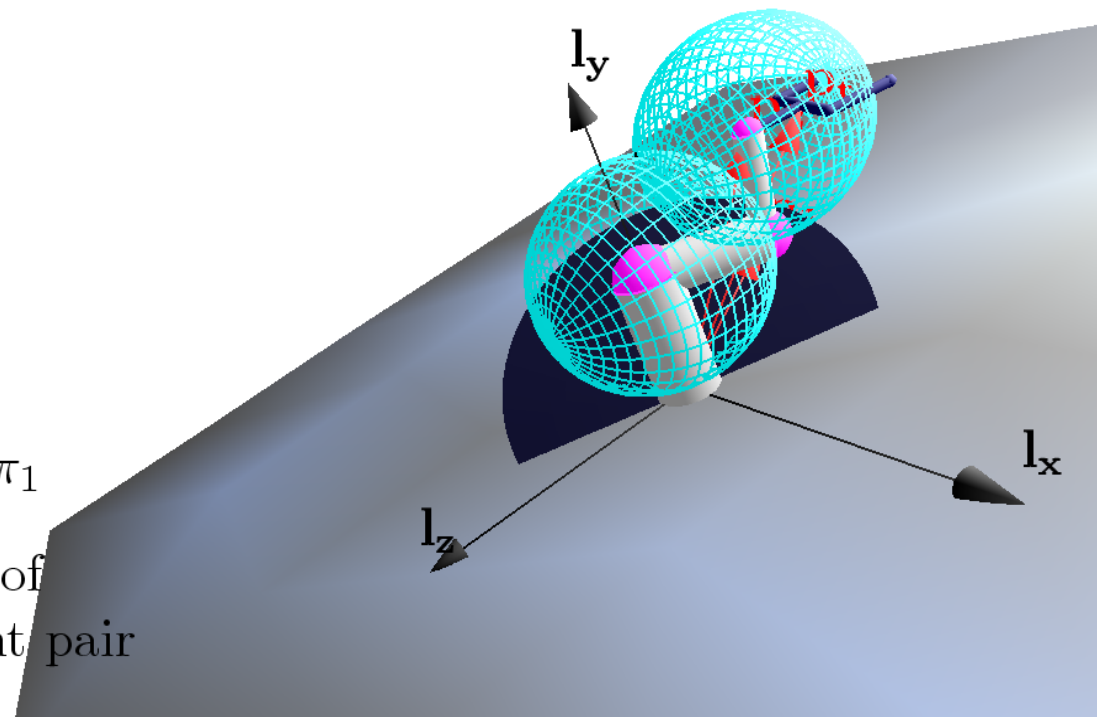
Inverse Kinematics Example



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Step 3 : compute p_1

- sphere around p_0
 $S_1 = p_0 - \frac{1}{2}d_2^2 e_\infty$
- sphere around p_2
 $S_2 = p_2 - \frac{1}{2}d_3^2 e_\infty$
- plane π_1
- $Pp_1 = S_1 \wedge S_2 \wedge \pi_1$
- choose one point of the resulting point pair



Inverse Kinematics Example



calculate the joint angles $\theta_1.. \theta_5$

• with $\angle(o_1, o_2) : \cos(\theta) = \frac{o_1^* \cdot o_2^*}{|o_1^*| |o_2^*|}$

– $\theta_1 = \angle(\pi_1, \pi_2)$

– $\theta_2 = \angle(l_1, l_y)$

– $\theta_3 = \angle(l_1, l_2)$

– $\theta_4 = \angle(l_2, l_3)$

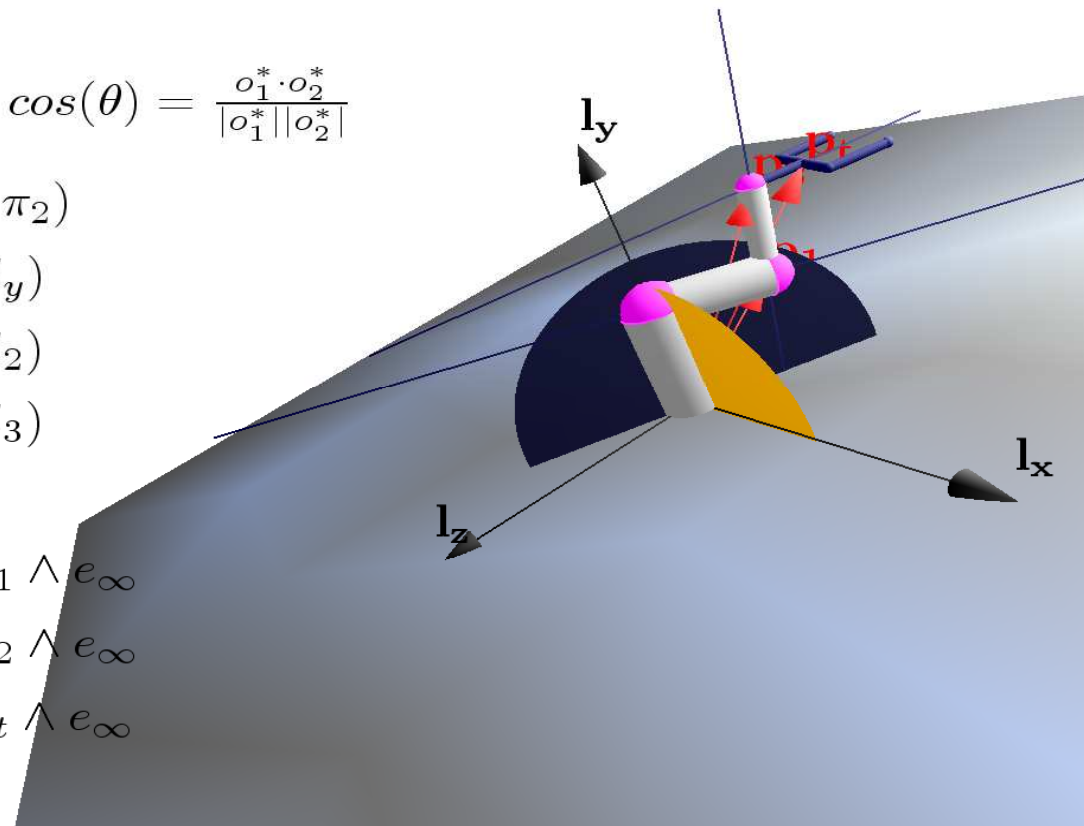
– with

– $l_1^* = p_0 \wedge p_1 \wedge e_\infty$

– $l_2^* = p_1 \wedge p_2 \wedge e_\infty$

– $l_3^* = p_2 \wedge p_t \wedge e_\infty$

– $\pi_2 = e_3$





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Thanks a lot ...